

# Problem-Solving Worksheet 10

## Algebraic Equations

Many of the relationships studied in chemistry involve algebraic equations. An algebraic equation is a string of letters and numbers with an equal sign in the middle. For example,  $4x + 7 = 15$  is an algebraic equation. To solve an algebraic equation for a variable (represented by a letter), you must move the variable to one side of the equal sign and move everything else to the other side. To accomplish this, you can add, subtract, multiply, divide, square, or perform another operation to one side of the equation, as long as you perform the same exact operation to the other side of the equation. Sometimes you will need to perform more than one operation to each side of the equation. In those cases, it is usually easier to add or subtract first, and then perform the other operations such as multiplying and dividing.

### Example 1

Solve for  $x$ :  $4x + 7 = 15$ .

#### Solution

First subtract 7 from both sides of the equation. This gives  $4x + 7 - 7 = 15 - 7$ , which is the same as  $4x = 8$ .

Now divide both sides of the equation by 4. (Note: Dividing by 4 is the same thing as multiplying by  $1/4$ .)

$$4x/4 = 8/4$$

$$x = 8/4$$

$$x = 2$$

Some equations contain more than one letter. In these cases, it is important to know which letter you are solving for.

### Example 2

Solve for  $y$ :  $\frac{3x - 10}{y} = 7$

#### Solution

First multiply both sides by  $y$ :  $\frac{3x - 10}{y} \times y = 7y$ , which is the same as  $3x - 10 = 7y$ .

Now divide both sides by 7, or multiply both sides by  $1/7$ .

$$\frac{3x - 10}{7} = \frac{7y}{7}$$

$$\frac{3x - 10}{7} = y$$

### Example 3

Solve for  $x$ :  $kx + 1 = my$ .

#### Solution

First subtract 1 from both sides:  $kx + 1 - 1 = my - 1$ , which is the same as  $kx = my - 1$ .

Now divide both sides by  $k$ .

$$\frac{kx}{k} = \frac{my - 1}{k}$$

$$x = \frac{my - 1}{k}$$

### Example 4

The density of a sample is the ratio of its mass to its volume. A sample of a substance has a mass of 15.0 g and a density of 2.80 g/cm<sup>3</sup>. Find the volume of the sample.

#### Solution

The formula for density is  $\text{density} = \frac{\text{mass}}{\text{volume}}$ . This gives the equation  $D = m/v$ . To solve this equation for  $v$ , first multiply both sides of the equation by  $v$ .

$$D \times v = \frac{m}{v} \times v$$

$$Dv = m$$

Now divide both sides by  $D$ .

$$\frac{Dv}{D} = \frac{m}{D}$$

$$v = m/D$$

Since  $m = 15.0$  g and  $D = 2.80$  g/cm<sup>3</sup>, you can substitute these numbers into the rearranged equation.

$$v = \frac{15.0 \text{ g}}{2.80 \text{ g/cm}^3} = \frac{15.0}{2.80} \frac{\text{g} \times \text{cm}^3}{\text{g}} = 5.36 \text{ cm}^3$$

### Exercises

1. Solve for  $x$ :  $14x + 12 = 40$ .

2. Solve for  $x$ :  $56/x = 22$ .

3. Solve for  $x$ :  $5/x + 8 = 11$ .

4. Solve for  $k$ :  $kx = a + by$

5. Solve for  $w$ :  $2l + 2w = 38$ .

6. Solve for  $z$ :  $\frac{y}{4z} = \frac{x}{w}$ .

7. Solve for  $x$ :  $3x_1 + 5y_1 = 2x_2 - 8y_2$ .

8. Solve for  $b$ :  $a + b + c = d$ .

9. Solve for  $v_2$ :  $l_1v_2 - 2k_2v_1 = 0$ .

10. A sample of a substance has a density of 3.20 g/cm<sup>3</sup> and a mass of 84.0 g. Find the volume of the sample.

# Problem-Solving Worksheet 5

## Calculating With Fractions

When something is divided into equal parts, the relation of one or more equal parts to the total of equal parts is known as a fraction. If a sample has a total mass of 10 g, then a 3-g portion of the sample is  $\frac{3}{10}$  of the sample. The top number of a fraction (3, in this case) is called the numerator, and the bottom number (10, in this case) is called the denominator.

### Equivalent Fractions

Two fractions are equivalent, or represent the same quantity, if you can multiply or divide both the numerator and the denominator of one of the fractions by the same number and end up with the other fraction. For example,  $\frac{2}{3}$  and  $\frac{10}{15}$  are equivalent fractions because  $\frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$ .

A fraction is in lowest terms if there is no number that can divide into both the numerator and the denominator. If a fraction is not in lowest terms, you can reduce it by dividing the numerator and the denominator by the largest number that divides evenly into both.

#### Example 1

Is  $\frac{24}{60}$  in lowest terms?

#### Solution

No,  $\frac{24}{60}$  can be reduced by dividing both 24 and 60 by 12. This gives  $\frac{2}{5}$ . Since no number divides into both 2 and 5,  $\frac{2}{5}$  is in lowest terms.

### Adding and Subtracting Fractions

In order to add or subtract two fractions, you must find a common denominator for the fractions. A common denominator is a number that is divisible by both of the denominators in the fractions you are adding or subtracting. Then add or subtract the numerators, keeping the common denominator unchanged.

#### Example 2

Add  $\frac{3}{5}$  to  $\frac{1}{8}$ .

#### Solution

First convert both  $\frac{3}{5}$  and  $\frac{1}{8}$  to equivalent fractions with the same denominator. Since 5 and 8 both divide into 40, 40 is a common denominator. Change  $\frac{3}{5}$  to  $\frac{24}{40}$  by multiplying the numerator and denominator by 8. Change  $\frac{1}{8}$  to  $\frac{5}{40}$  by multiplying the numerator and denominator by 5. The answer can then be computed as follows:  $\frac{24}{40} + \frac{5}{40} = \frac{29}{40}$ .

### Multiplying Fractions

To multiply two fractions, first multiply the numerators; then multiply the denominators

#### Example 3

Multiply  $\frac{9}{10} \times \frac{2}{3}$ .

#### Solution

First multiply the numerators ( $9 \times 2 = 18$ ); then multiply the denominators ( $10 \times 3 = 30$ ). The answer is  $\frac{18}{30}$  or  $\frac{3}{5}$ .

## Problem Solving Worksheet 5 (continued)

### Dividing Fractions

To divide one fraction by another, invert the fraction in the denominator, then multiply it by the fraction in the numerator.

#### Example 4

Divide  $\frac{6}{13}$  by  $\frac{4}{7}$ .

#### Solution

First invert  $\frac{4}{7}$ ; this gives  $\frac{7}{4}$ . Now multiply  $\frac{6}{13}$  by  $\frac{7}{4}$ .  
 $\frac{6}{13} \times \frac{7}{4} = \frac{42}{52}$  or  $\frac{21}{26}$ .

### Converting Fractions to Decimals

To change a fraction to a decimal, divide the numerator by the denominator.

#### Example 5

Convert  $\frac{7}{8}$  to a decimal.

#### Solution

$$\begin{array}{r} 0.875 \\ 8 \overline{)7.000} \\ \underline{64} \phantom{00} \\ 60 \phantom{0} \\ \underline{56} \phantom{0} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

The answer is 0.875.

### Exercises

1. Add  $\frac{2}{3}$  to  $\frac{5}{12}$ .

6. Add  $\frac{26}{15}$  to  $\frac{4}{9}$ .

2. Subtract  $\frac{1}{2}$  from  $\frac{7}{6}$ .

7. Subtract  $\frac{11}{20}$  from  $\frac{3}{4}$ .

3. Multiply  $\frac{14}{3}$  by  $\frac{7}{8}$ .

8. Multiply  $\frac{5}{27}$  by  $\frac{3}{50}$ .

4. Divide  $\frac{5}{21}$  by  $\frac{15}{14}$ .

9. Divide  $\frac{4}{25}$  by  $\frac{18}{5}$ .

5. Convert  $\frac{7}{8}$  to a decimal. Round to the nearest hundredth.

10. Convert  $\frac{5}{16}$  to a decimal.